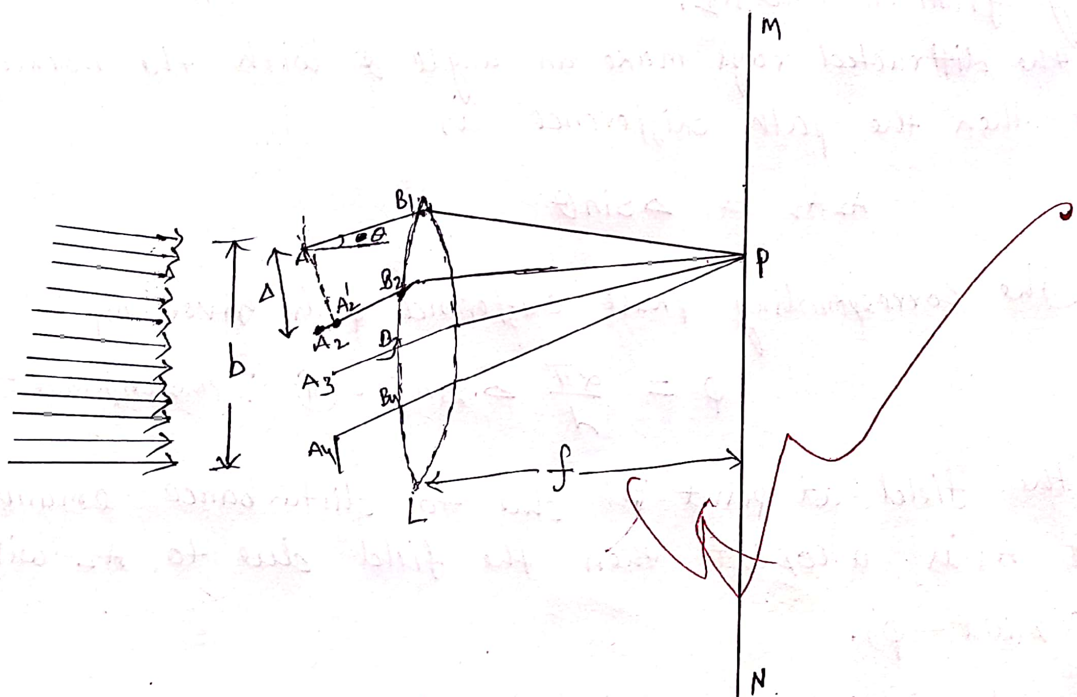
Diffraction :- Diffraction is the bending or spreading of a beam of light when passing through a narrow opening or by the edge of an object.

Conditions for diffraction :- (i) Source of light should be monochromatic

(ii) wavelength of light used should be comparable to the size of the obstacle.

Fraunhofer Diffraction due to single slit :- In Fraunhofer diffraction the source and the screen are a infinite distance from the obstacle and the wavefront is plane.



Let us consider a long narrow slit of width 'b'. $A_1, A_2, A_3 \cdots$ are the point sources and 'Δ' be the distance between two consecutive points.

If the number of point sources is n, then

$$b = (n-1)\Delta \quad \text{--- ①}$$

$$\Delta = A_2 - A_1$$

$$b = (n-1)\Delta$$

if $n \to \infty$, $\Delta \to 0$

$$\boxed{b = n\Delta}$$

b is the order of milimeter (mm). But the distance between slit to screen of the order of meter, due to this all the waves emanating from secondary wavelets are of same magnitude of amplitude and different phase difference.

From the figure A_1A_1' is the path difference between the waves coming from A_1 and A_2.

If the diffracted rays make an angle θ with the normal to the slit then the path difference is

$$A_2A_2' = \Delta \sin\theta$$

The corresponding phase difference ϕ is given by

$$\phi = \frac{2\pi}{\lambda}\Delta \sin\theta \quad \text{(II)} \quad \left(\because \text{phase difference} = \frac{2\pi}{\lambda}\text{path di}\right)$$

If the field at point 'P' due to disturbance emanating from Point A_1 is $a\cos\omega t$ then the field due to A_2 will be $a\cos(\omega t - \phi)$.

The resultant field (E) at point 'P' will be

$$E = a\cos\omega t + a\cos(\omega t - \phi) + \cdots a\cos[\omega t - (n-1)\phi] \quad \text{(III)}$$

$$E = \left(\frac{a \sin\frac{n\phi}{2}}{\sin\frac{\phi}{2}}\right)\cos\left[\omega t - \frac{(n-1)\phi}{2}\right]$$

$$E = E_0 \cos\left[\omega t - \frac{(n-1)\phi}{2}\right] \quad \text{(IV)}$$

where $E_0 = \dfrac{a \sin \frac{n\phi}{2}}{\sin \frac{\phi}{2}}$

If $n \to \infty$, $\Delta \to 0$

and $b = n\Delta$

then

$$\frac{n\phi}{2} = \frac{\pi}{\lambda} n\Delta \sin\theta$$

$$\frac{n\phi}{2} = \frac{\pi}{\lambda} b\sin\theta$$

$$\phi = \frac{2\pi}{n\lambda} b\sin\theta$$

$$E_0 = \frac{a \sin \frac{n\phi}{2}}{\sin \frac{\phi}{2}} = a \frac{\sin\left(\frac{\pi}{\lambda} b\sin\theta\right)}{\sin\frac{\phi}{2}}$$

$\because$ $\sin\frac{\phi}{2} \simeq \frac{\phi}{2}$. ($\because$ ϕ is small)

$$E_0 = a \frac{\sin\left(\frac{\pi}{\lambda} b\sin\theta\right)}{\frac{\phi}{2}}$$

$$E_0 = \frac{a \sin\left(\frac{\pi}{\lambda} b\sin\theta\right)}{\frac{\pi}{n\lambda} b\sin\theta}$$

let $\frac{\pi}{\lambda} b\sin\theta = \beta$

Then $E_0 = \dfrac{a \sin\beta}{\frac{1}{n}\beta}$

$$\boxed{E_0 = na \frac{\sin\beta}{\beta}}$$

Hence

$$E = E_0 \cos\left[\omega t - \frac{(n-1)}{2}\phi\right]$$

$$E = na \frac{\sin\beta}{\beta} \cos\left[\omega t - \frac{(n-1)}{2}\phi\right] \quad \text{— (V)}$$

$$E = na \frac{\sin\beta}{\beta} \cos\left[\omega t - \frac{n\phi}{2} + \frac{\phi}{2}\right]$$

Let $na = A$ (Amplitude of all wavelets)

$$E = A \frac{\sin\beta}{\beta} \cos\left[\omega t - \frac{1}{2}\frac{2\pi}{\lambda} b\sin\theta + \frac{\pi b}{n\lambda}\sin\theta\right]$$

[if $n \to \infty$ then $\frac{\pi b}{n\lambda}\sin\theta \simeq 0$]

$$E = A \frac{\sin\beta}{\beta} \cos\left[\omega t - \frac{\pi}{\lambda} b\sin\theta\right]$$

$$\boxed{E = A \frac{\sin\beta}{\beta} \cos(\omega t - \beta)}$$

$\left(\because \beta = \frac{\pi}{\lambda} b\sin\theta\right)$

where $A\frac{\sin\beta}{\beta}$ is the amplitude of resultant wave.

then the intensity I will be

$$I = (\text{amplitude})^2$$

$$I = A^2 \frac{\sin^2\beta}{\beta^2}$$

$$\boxed{I = I_0 \frac{\sin^2\beta}{\beta^2}}$$

(where $I_0 = A^2$)

I_0 is the intensity at $\theta = 0$.

(Remaining part of this topic will be discussed in next class)